

Visual Analytics for discovering group structures in networks

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Exploratory Analysis in Networks

Most network analyses try to detect a **given type of structure**, such as

- Small-world

Watts & Strogatz, Nature **393**, 440 (1998),

- Scale-free

Barabási & Albert, Science **286**, 509 (1999),

- Community structure

Newman & Girvan, PRE **69**, 026113 (2004),

Porter, Onnela, & Mucha, Notices Amer Math Soc **56**, 1082 (2009),

....

Exploratory Analysis in Networks

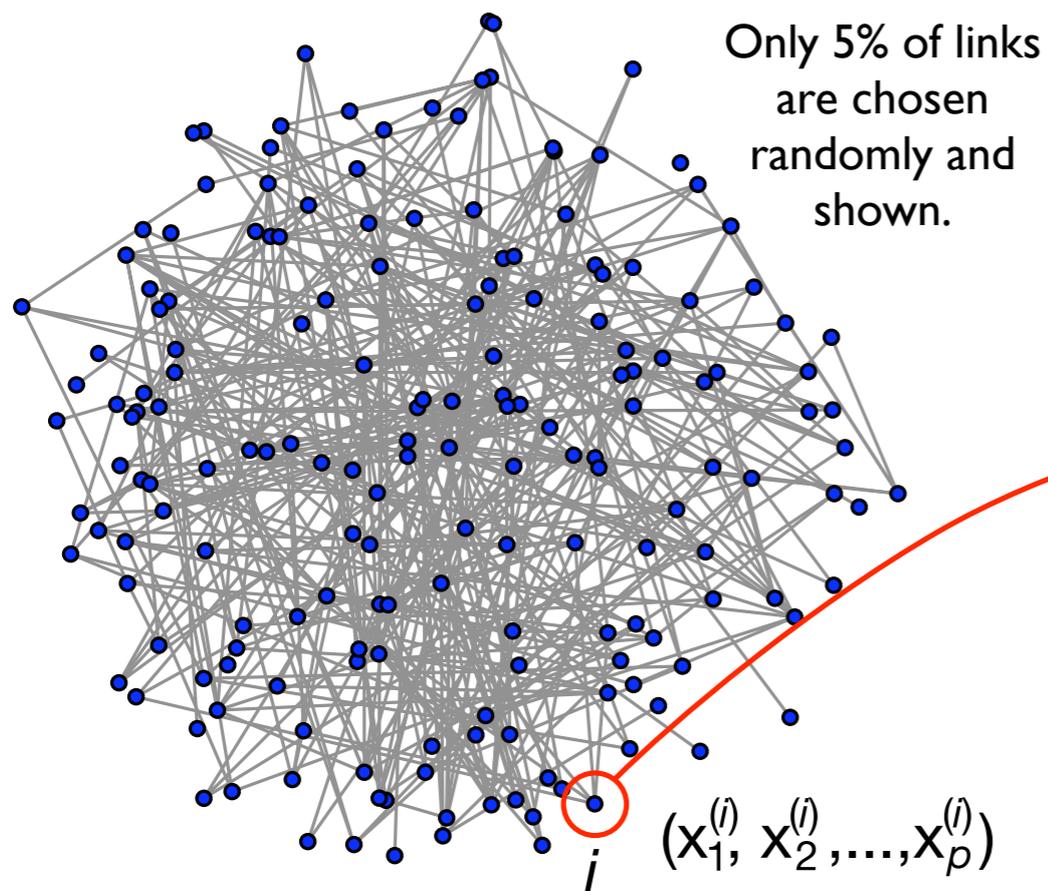
- An exploratory analysis tries to **discover an unknown structure**.
Newman & Leicht, PNAS **104**, 9564 (2007)
- We take visual analytics approach for this problem.

Visual Analytics

- Visual interaction between the **human user** and the **analytical tools** for analyzing and interpreting high-dimensional complex data
- We take this approach for discovering an unknown group structure among nodes in a network data set.

Discovering Unknown Group Structures

Ungrouped network



R^p

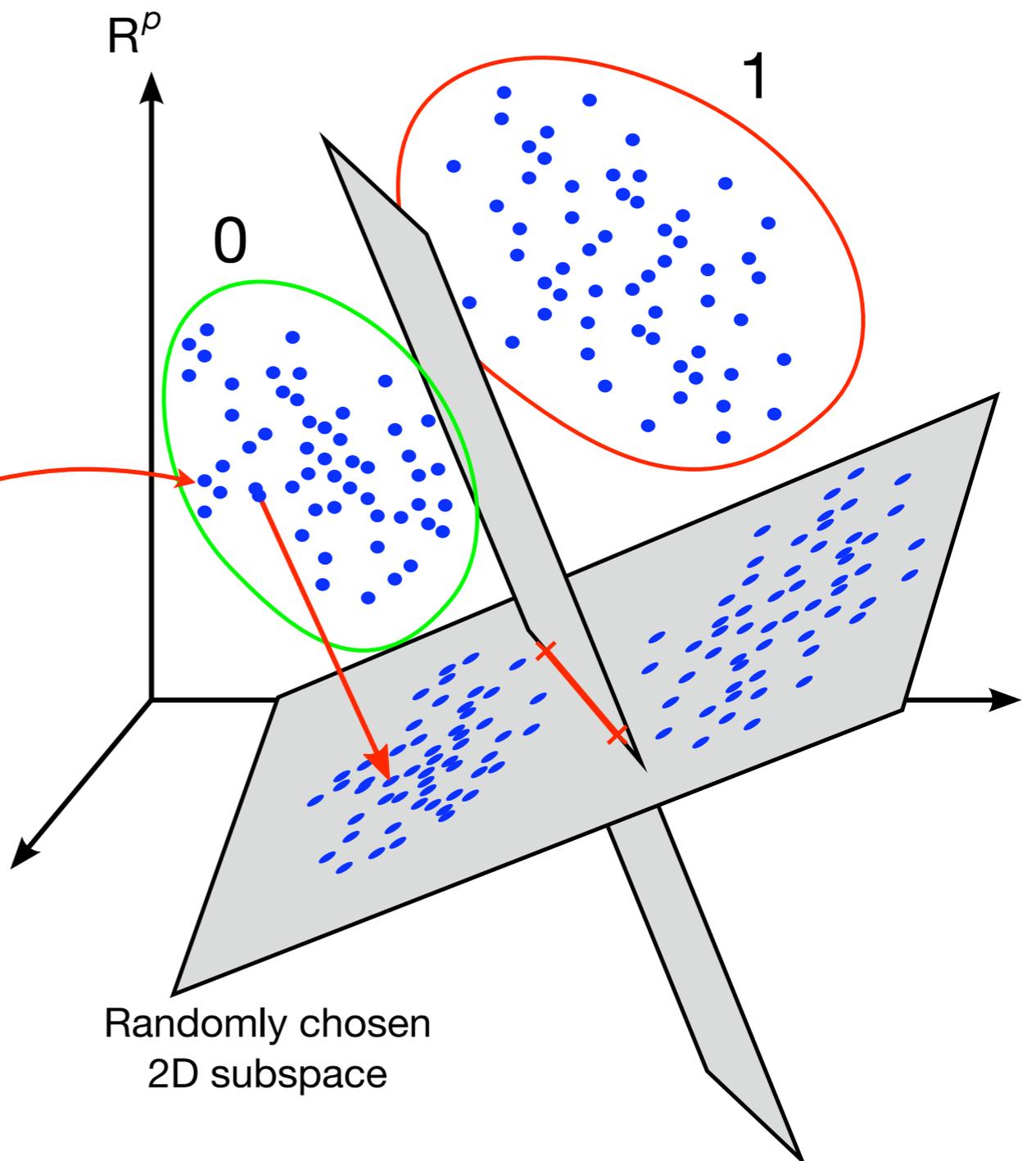
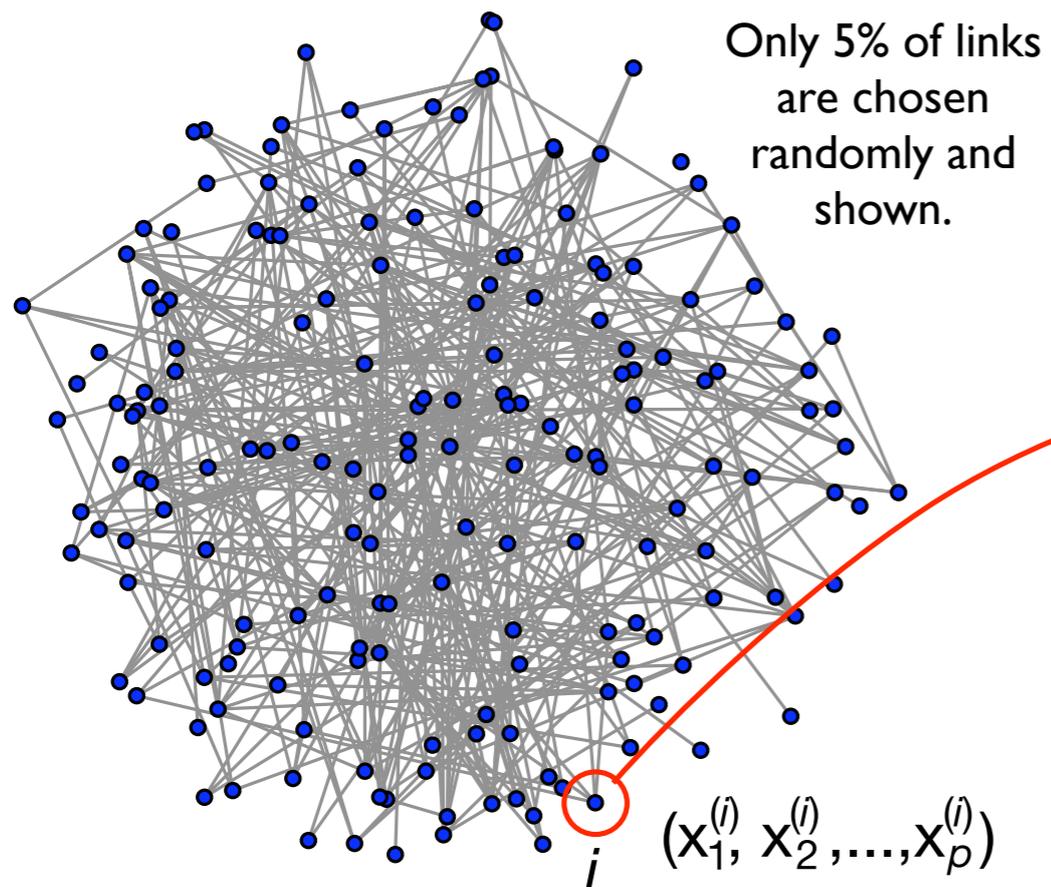
Distinct groups?

Degree
Neighbor's mean degree
2nd neighbor's mean degree
Clustering coefficient

Betweenness centrality
Laplacian eigenvectors
Normalized Laplacian eigenvectors
Mean shortest path

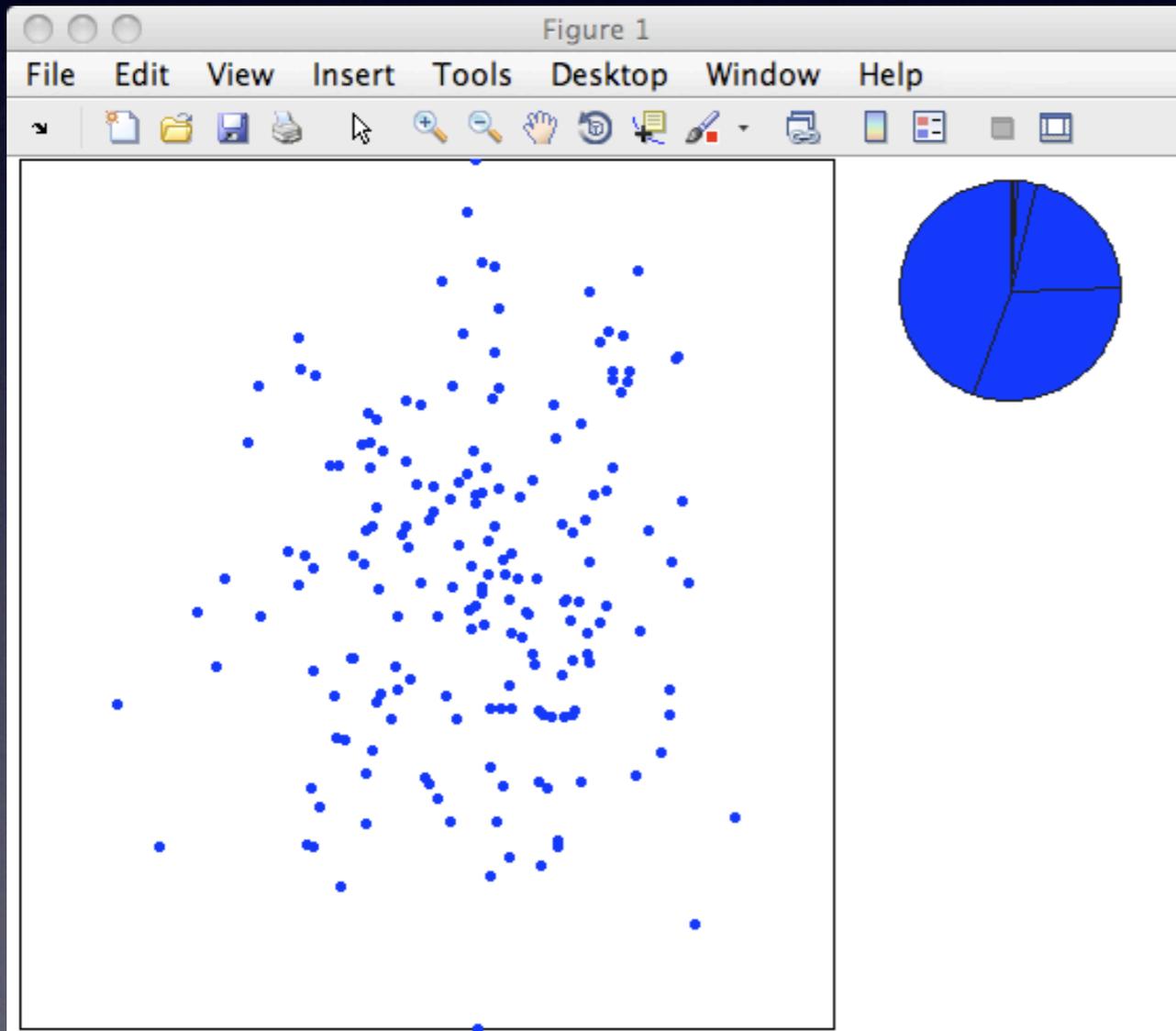
How it works: Random 2D Projection

Ungrouped network

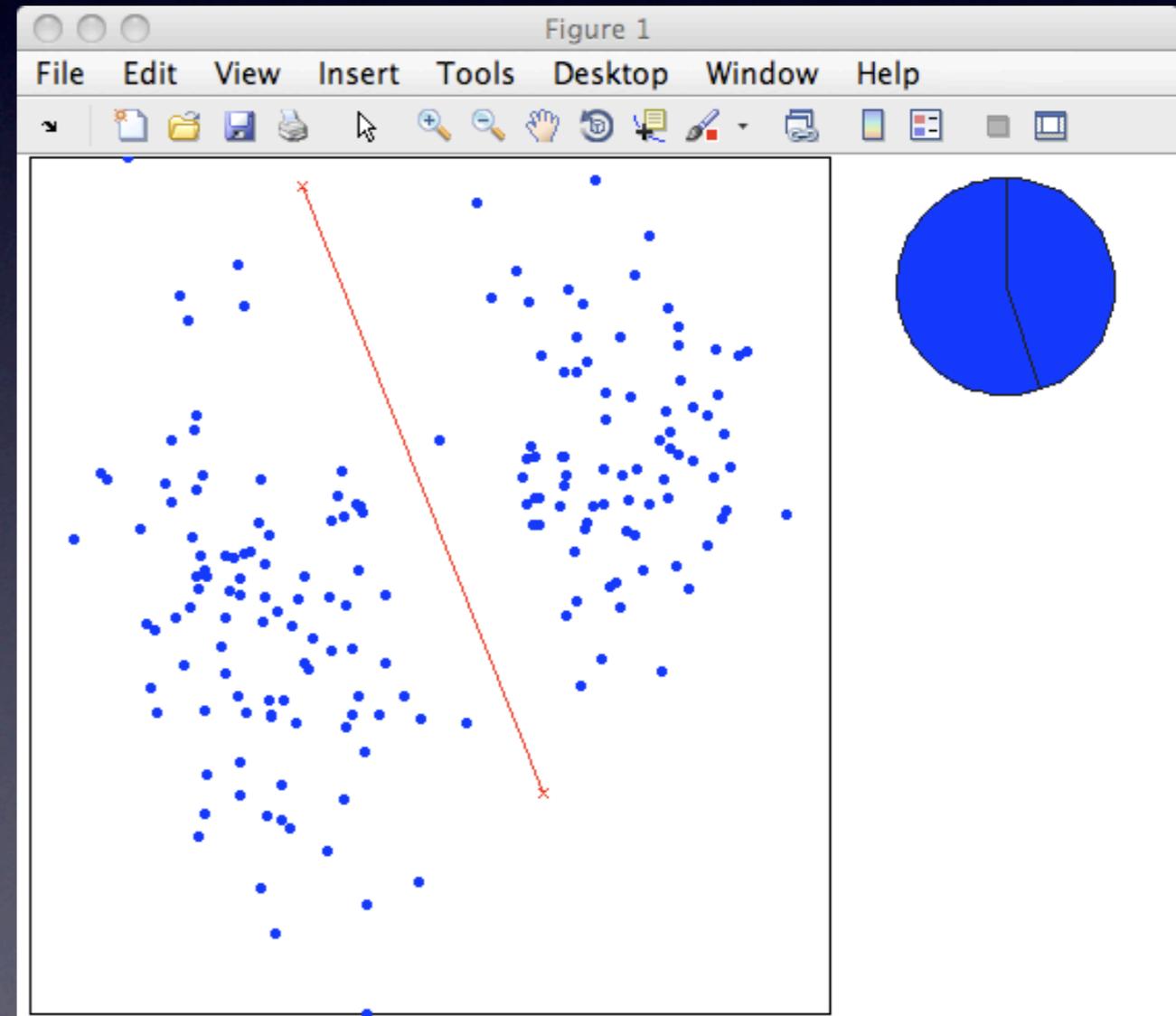


How it works: Visual Interactions

Reject



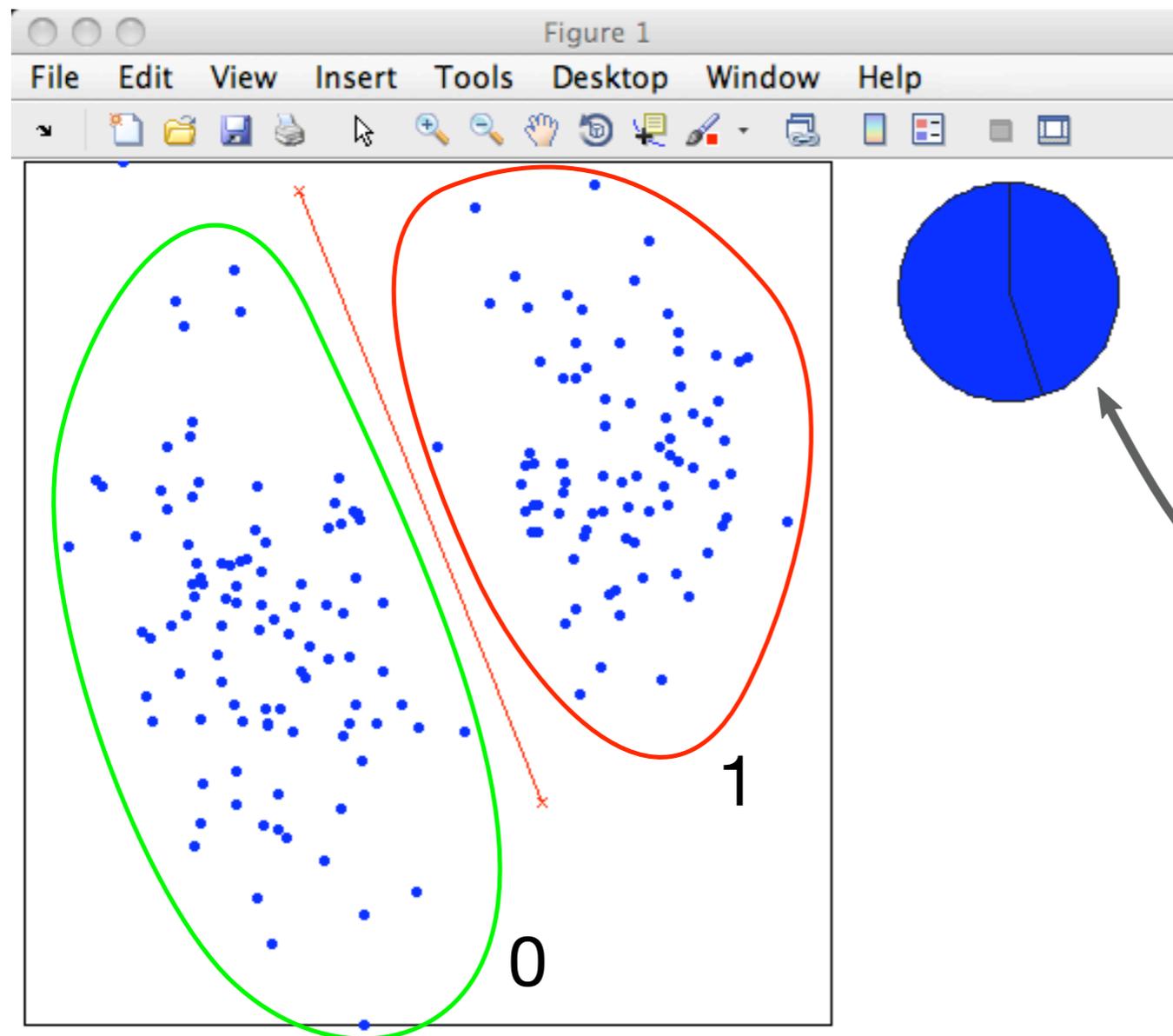
Divide



How it works: Patterns of User Inputs

Divide

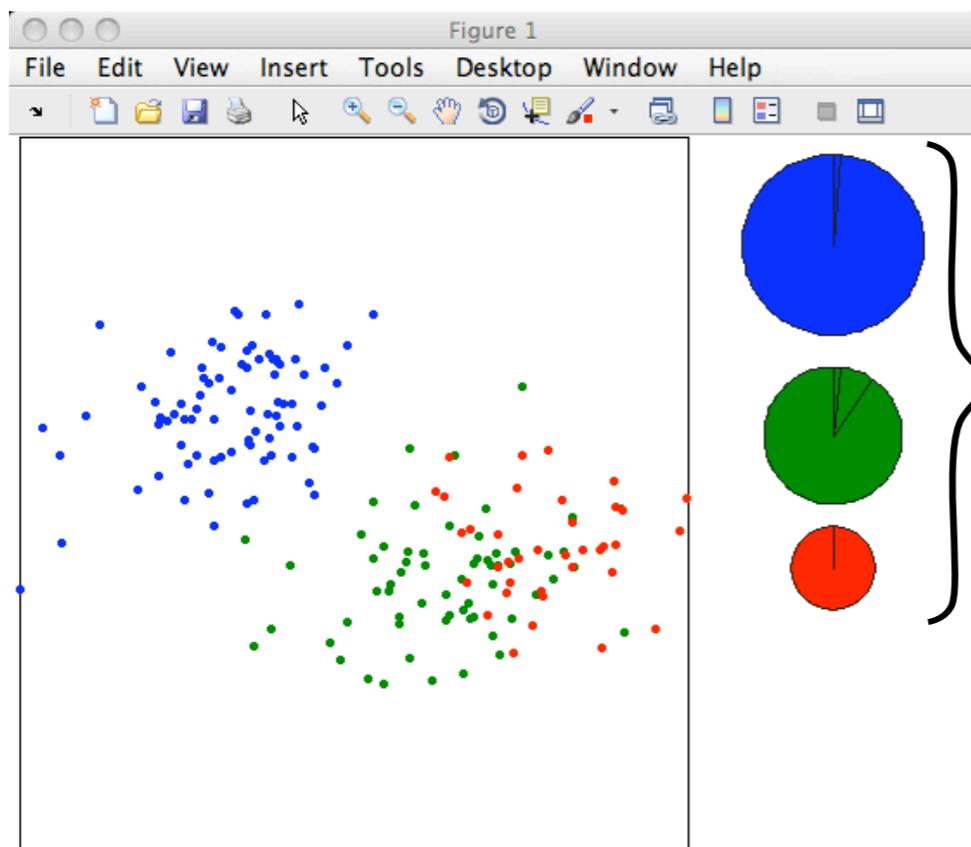
Binary patterns of divisions



Data point	Projection			
	#1	#2	#3	#4
#1	0	0	0	0
#2	0	1	0	0
#3	0	1	0	1
#4	0	0	0	0
#5	1	1	1	1
#6	1	1	0	1
#7	1	1	0	0
#8	1	1	1	1
#9	1	1	0	1
:	:	:	:	:
:	:	:	:	:

How it works: Grouping Binary Patterns

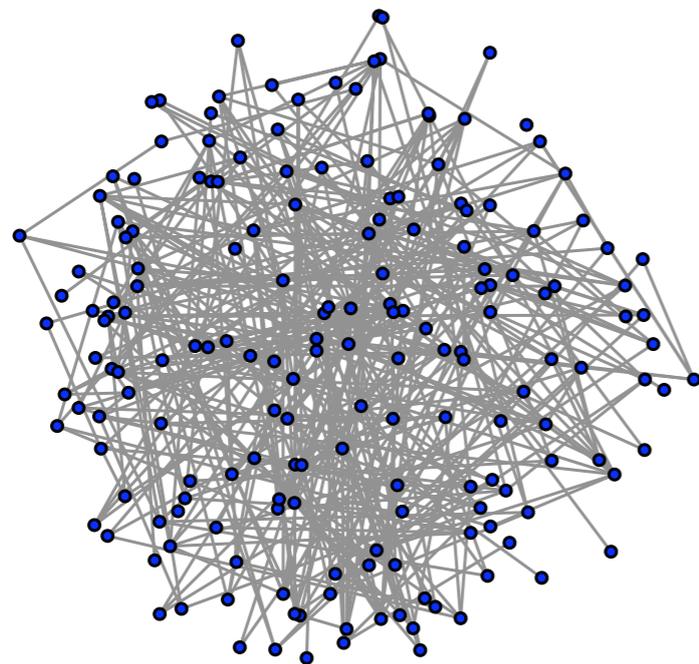
Choose number of groups



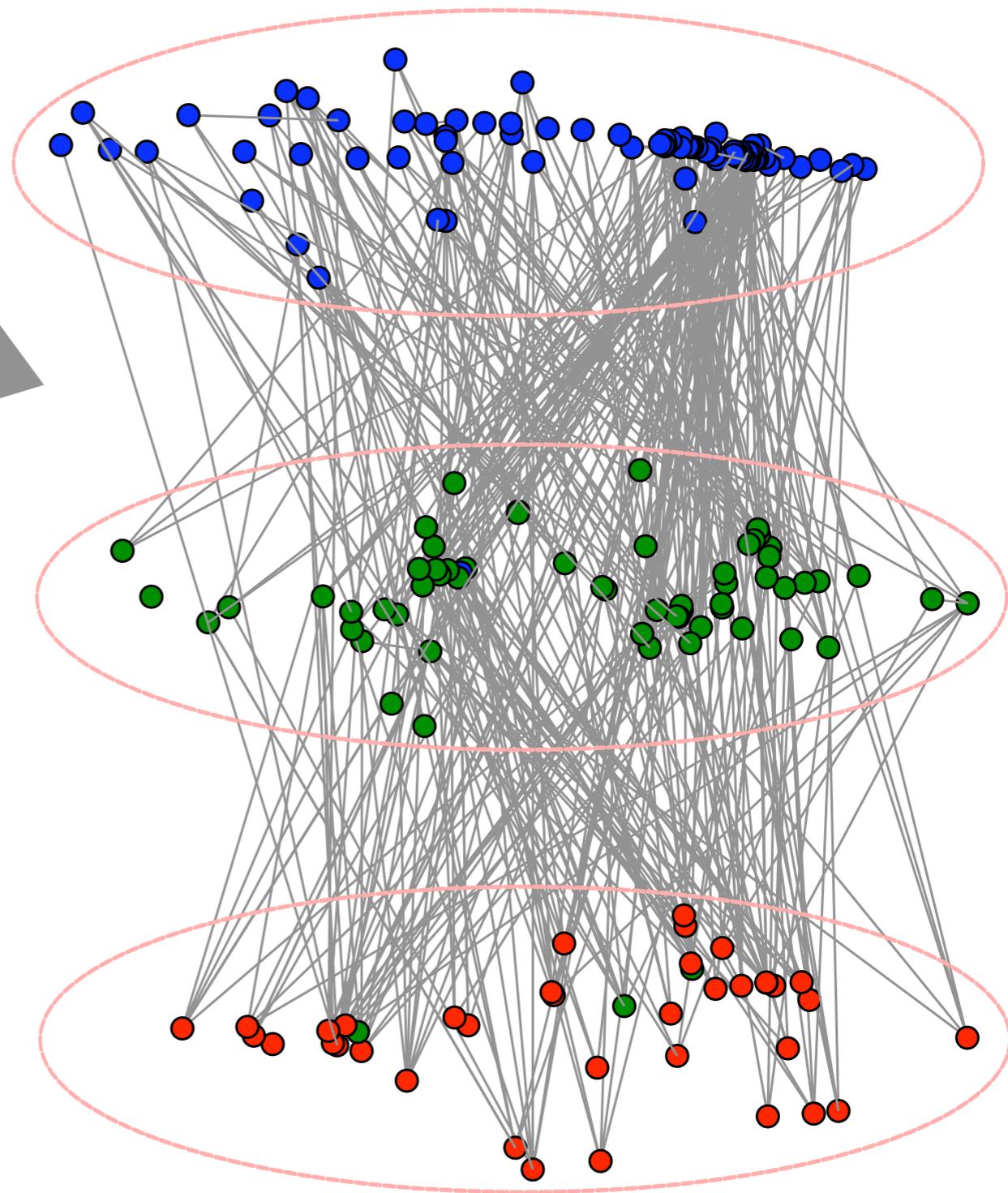
Grouping of binary pattern

Group	Principal pattern	Other patterns
#1	0000	0100
#2	1111	N/A
#3	1101	1100, 0101

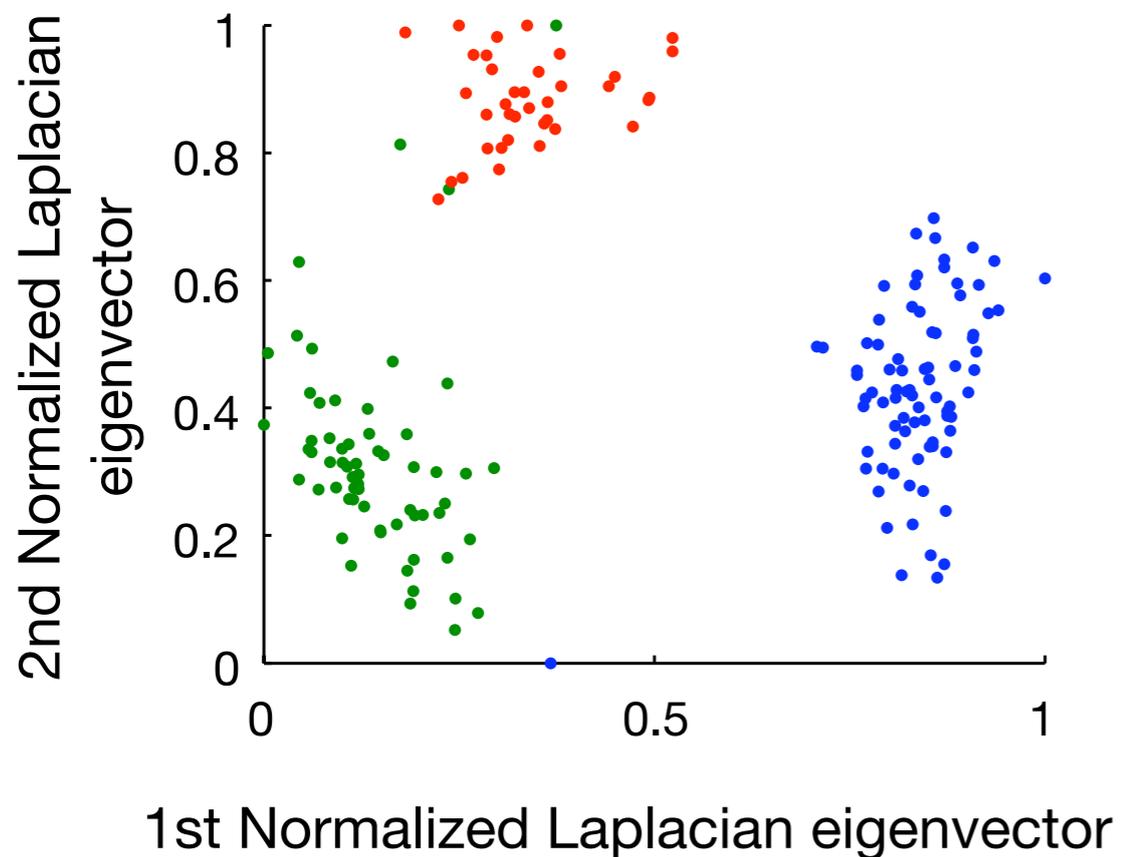
Ungrouped network



Grouped network



Discriminating node properties



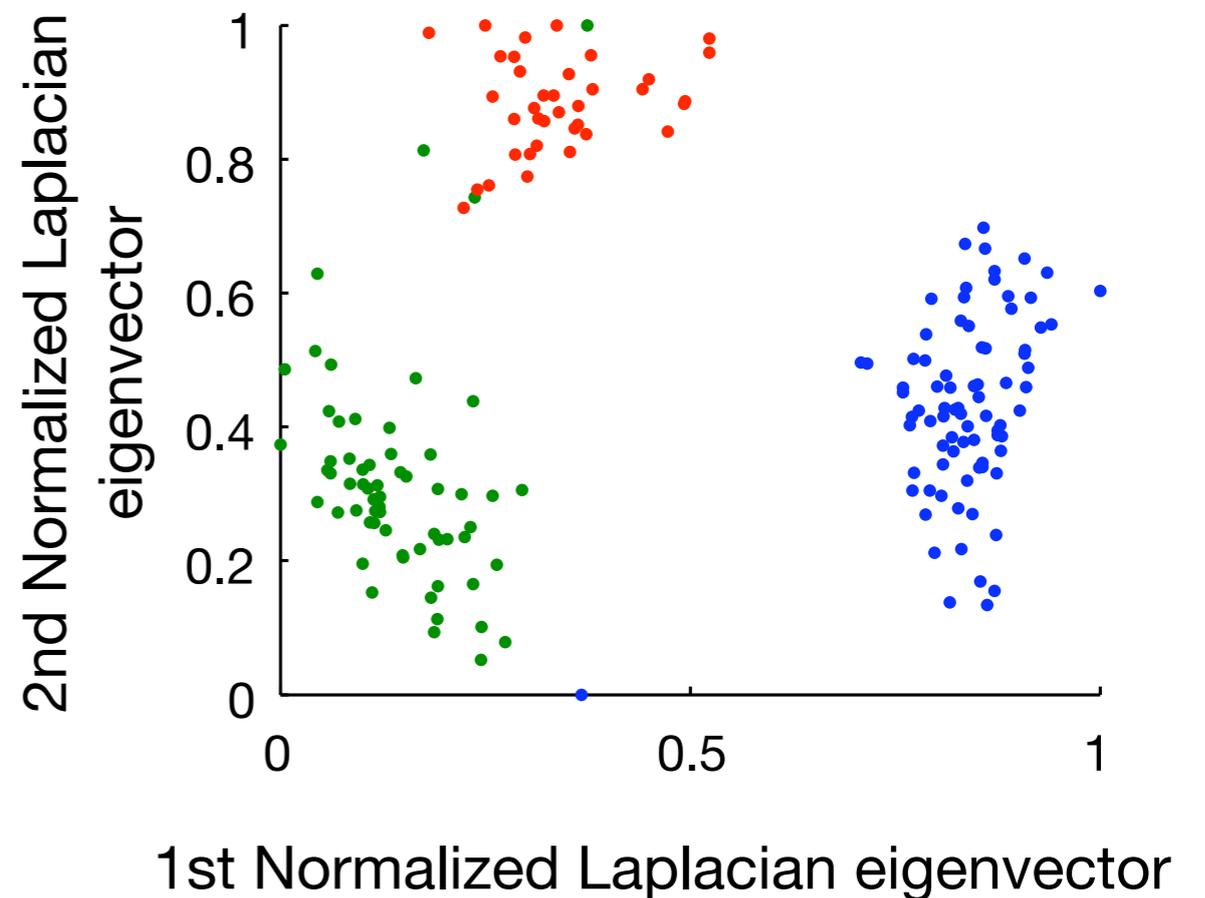
Colors - discovered assignment
Ellipses - prescribed assignment

Properties Discriminating Nodes

For node property j , define

$$D_j = \sum_{k=1}^K \sum_{k'=1}^K |\bar{x}_j^{(k)} - \bar{x}_j^{(k')}|$$

$$\bar{x}_j^{(k)} = \frac{1}{|G_k|} \sum_{i \in G_k} x_j^{(i)}$$



G_k : set of indices for nodes in group k

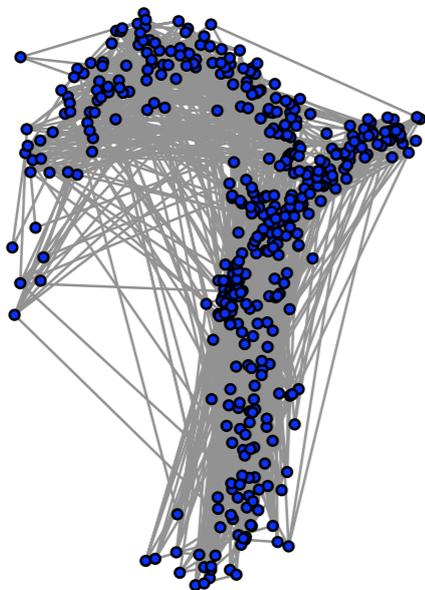
$|G_k|$ = the number of nodes in group k

We choose node properties with largest D_j , and project points onto the corresponding subspace.

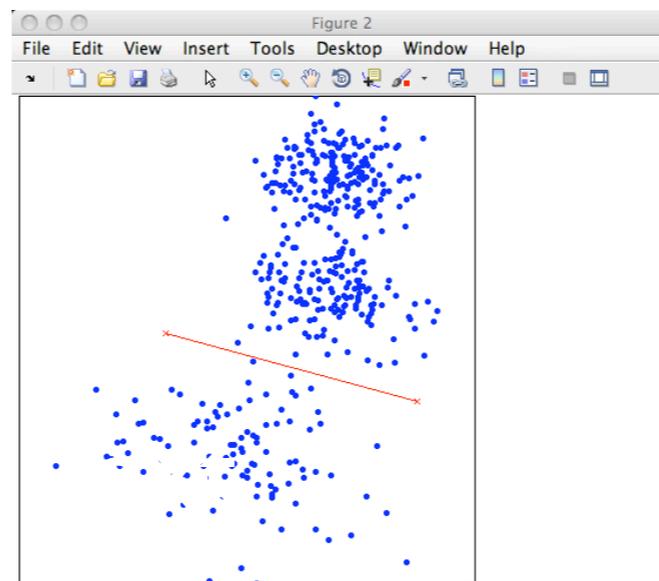
Example 2

Community Structure

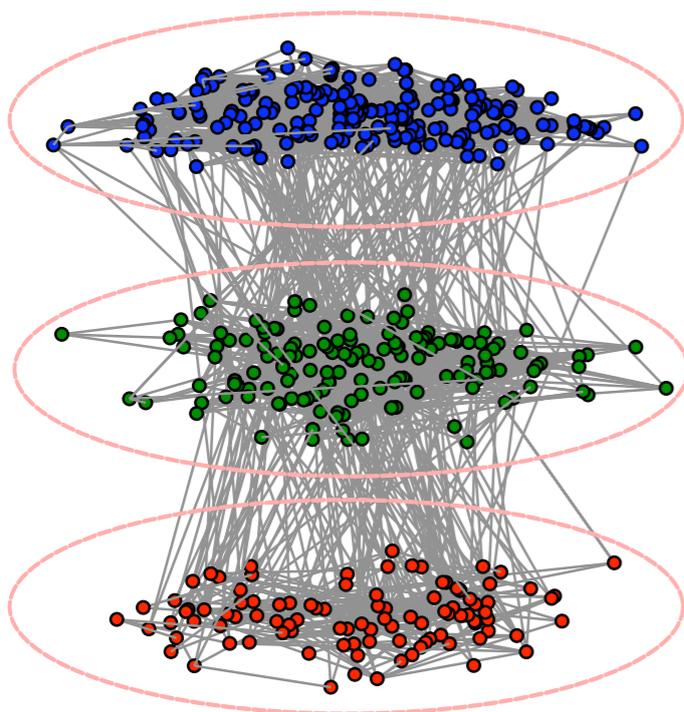
Ungrouped network



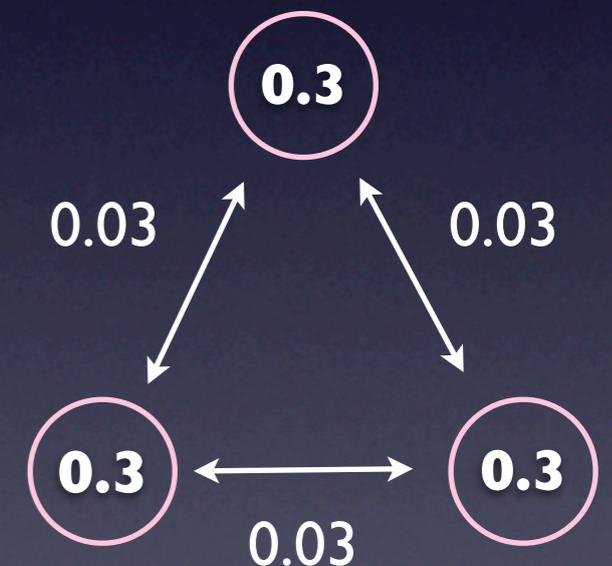
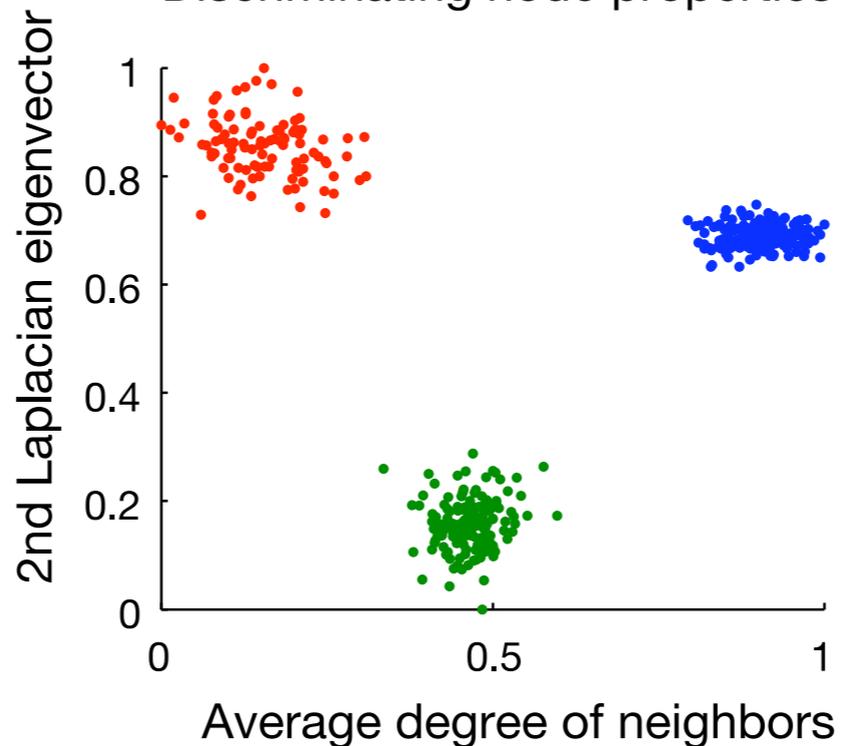
Divide



Grouped network



Discriminating node properties



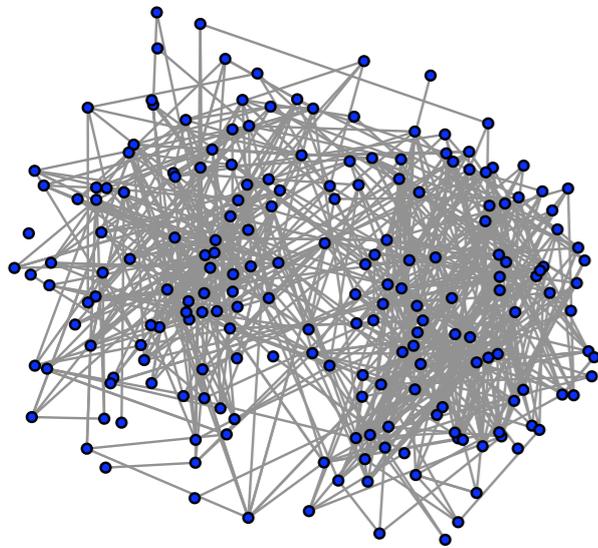
$$p_{\text{within}} = 0.3$$

$$p_{\text{across}} = 0.03$$

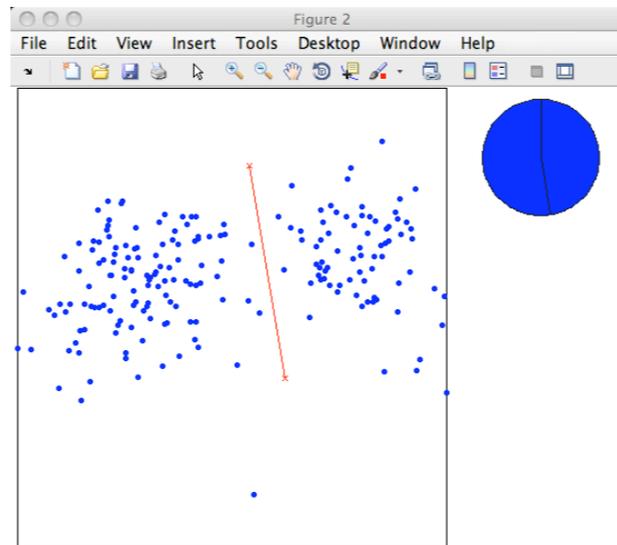
Example 3

Mixed Group Structure

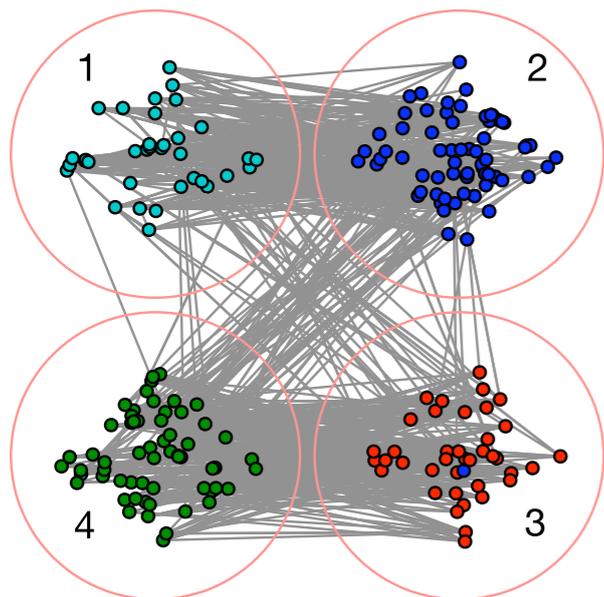
Ungrouped network



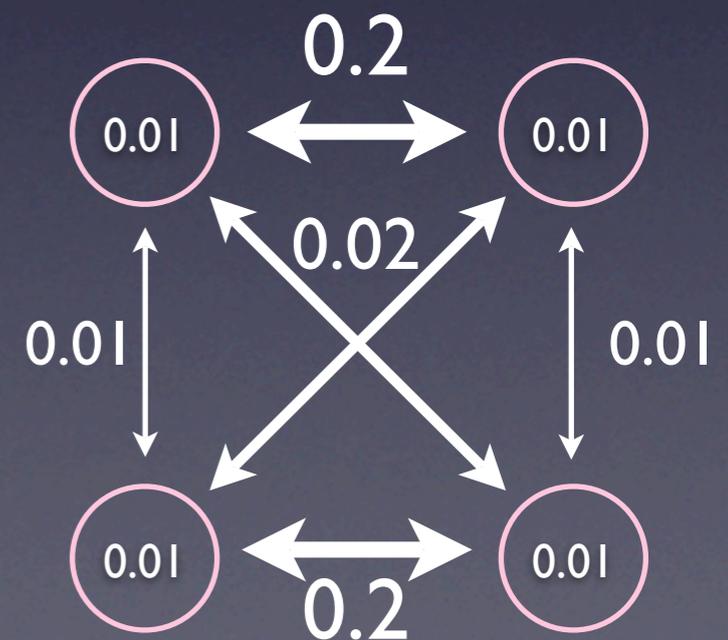
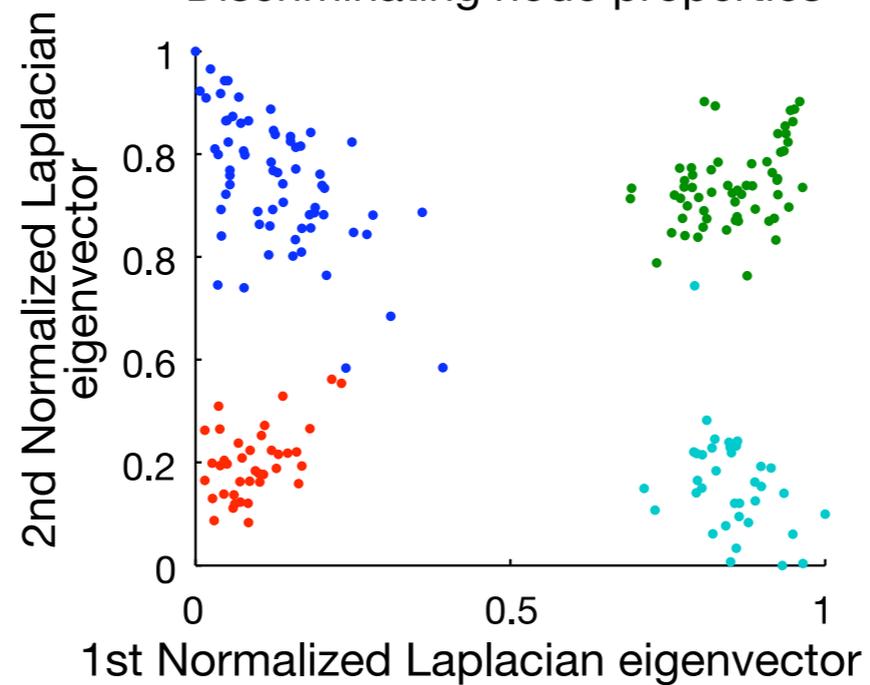
Divide



Grouped network



Discriminating node properties



Example 4: PACS Network

RAPID COMMUNICATIONS

PHYSICAL REVIEW E 73, 065106(R) (2006)

Synchronization is optimal in nondiagonalizable networks

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(2006)

PACS number(s): 89.75.-k, 05.45.Xt, 87.18.Sn

by assigning weights and directionality formalism to all possible networks, we show that maximally synchronizable networks are necessarily nondiagonalizable and can always be obtained by imposing unidirectional information flow with normalized input strengths. The results provide insights into hierarchical structures observed in complex networks in which synchronization is important.

DOI: [10.1103/PhysRevE.73.065106](https://doi.org/10.1103/PhysRevE.73.065106)

PACS number(s): 89.75.-k, 05.45.Xt, 87.18.Sn

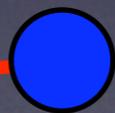
Under extensive study in recent years is how the collective dynamics of a complex network is influenced by the structural properties of the network [1], such as clustering coefficient [2], average network distance [3], connectivity

the network dynamics can be linearly decomposed into eigenmodes, i.e., the coupling matrix of the network is diagonalizable. Indeed, we show that maximally synchronizable networks are always *nondiagonalizable* (except for the ex-



89.75

Complex systems

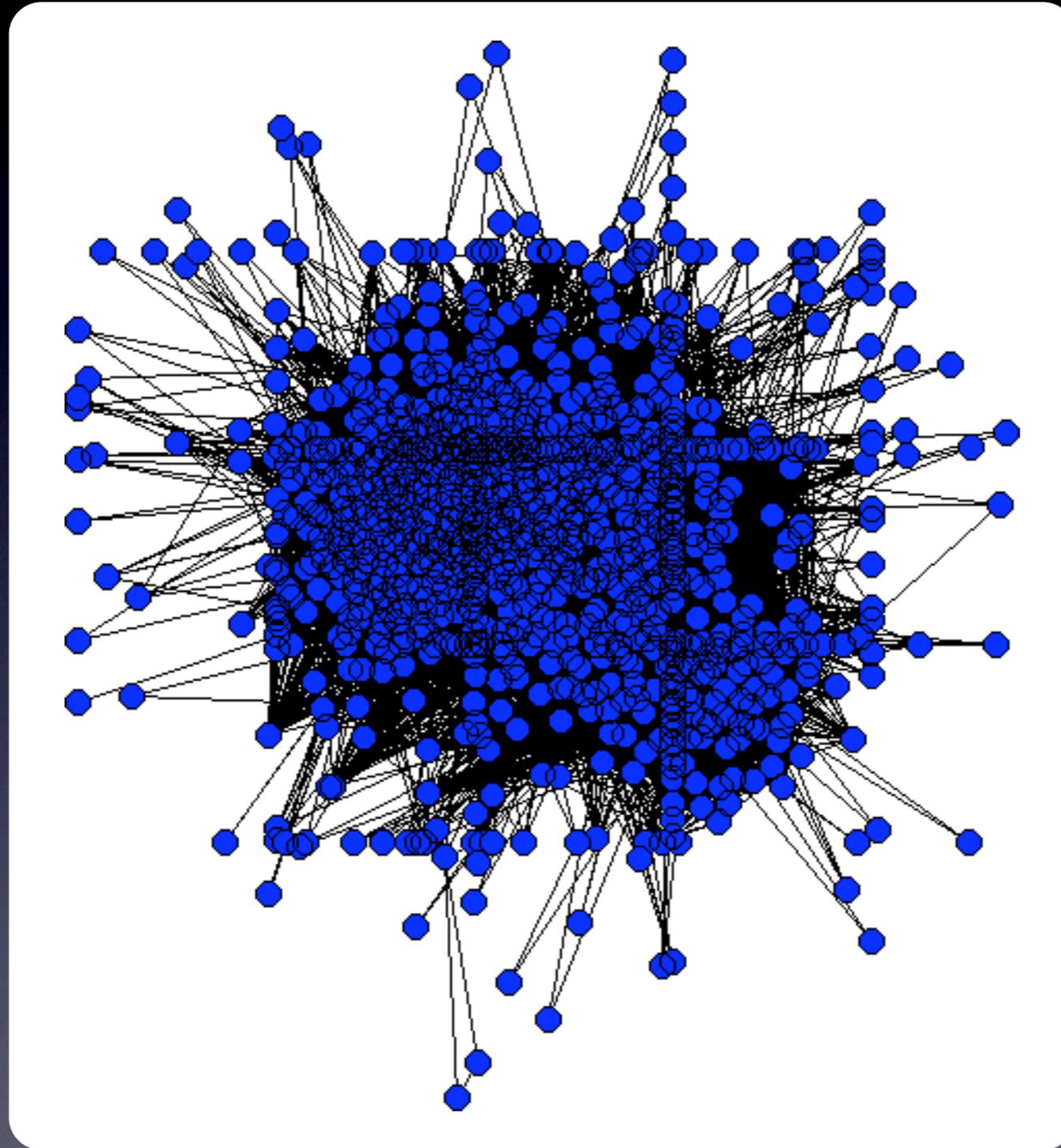


05.45

Nonlinear dynamics
and chaos

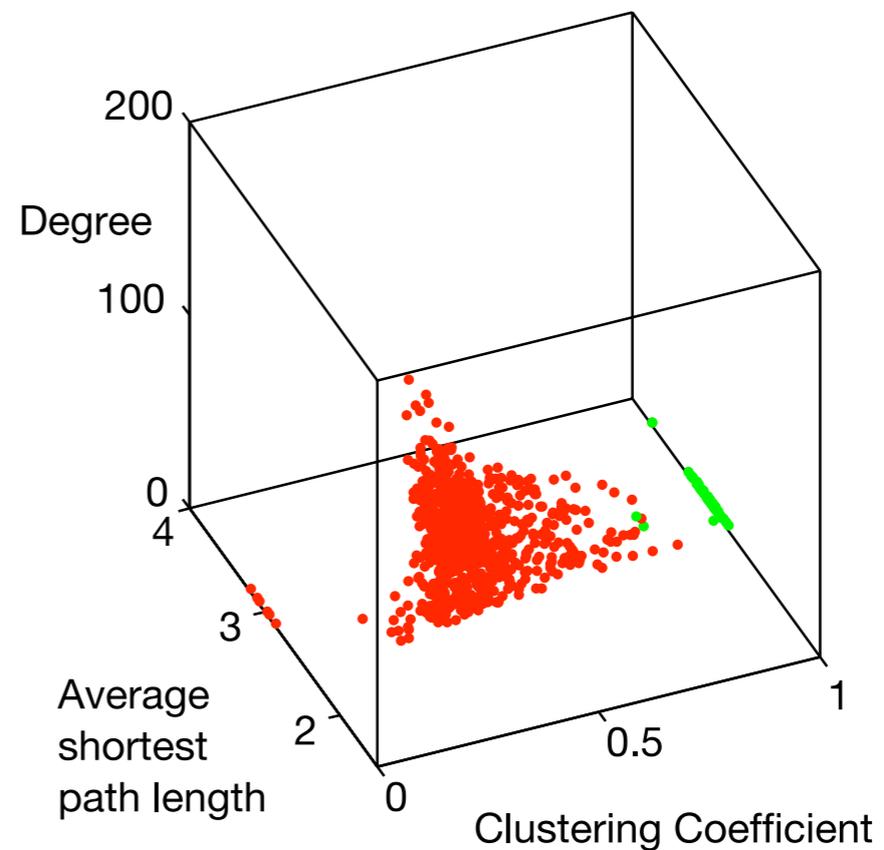
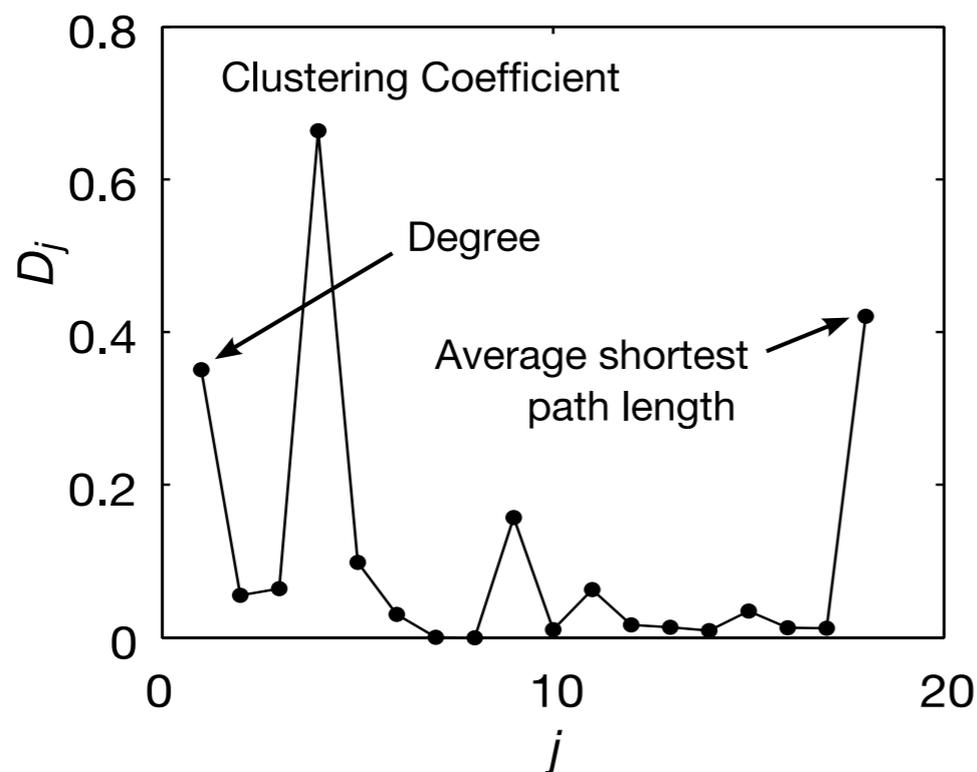
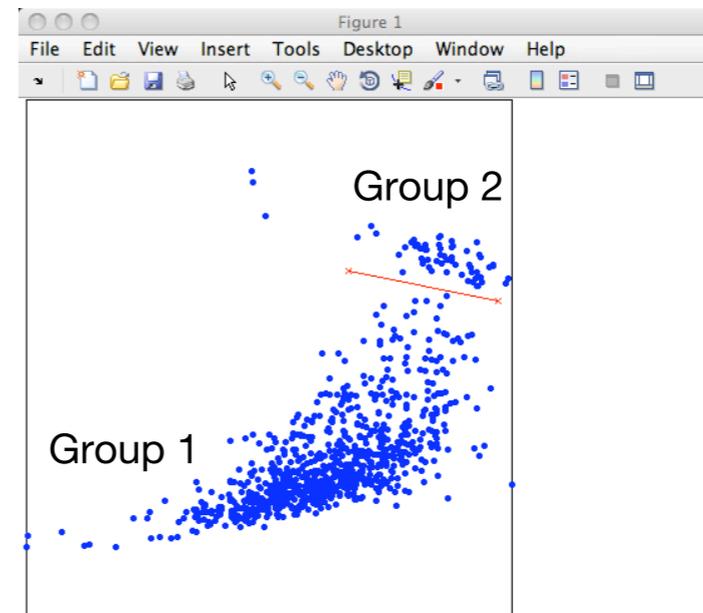
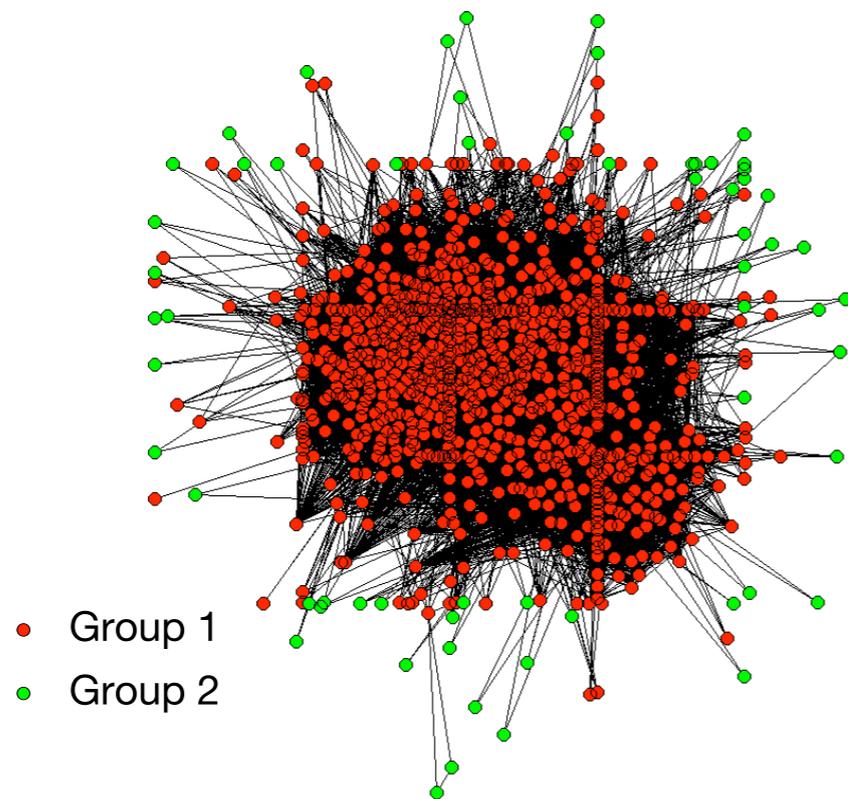
Nodes are connected if there are more papers than expected by random & independent occurrence

Example 4: PACS Network

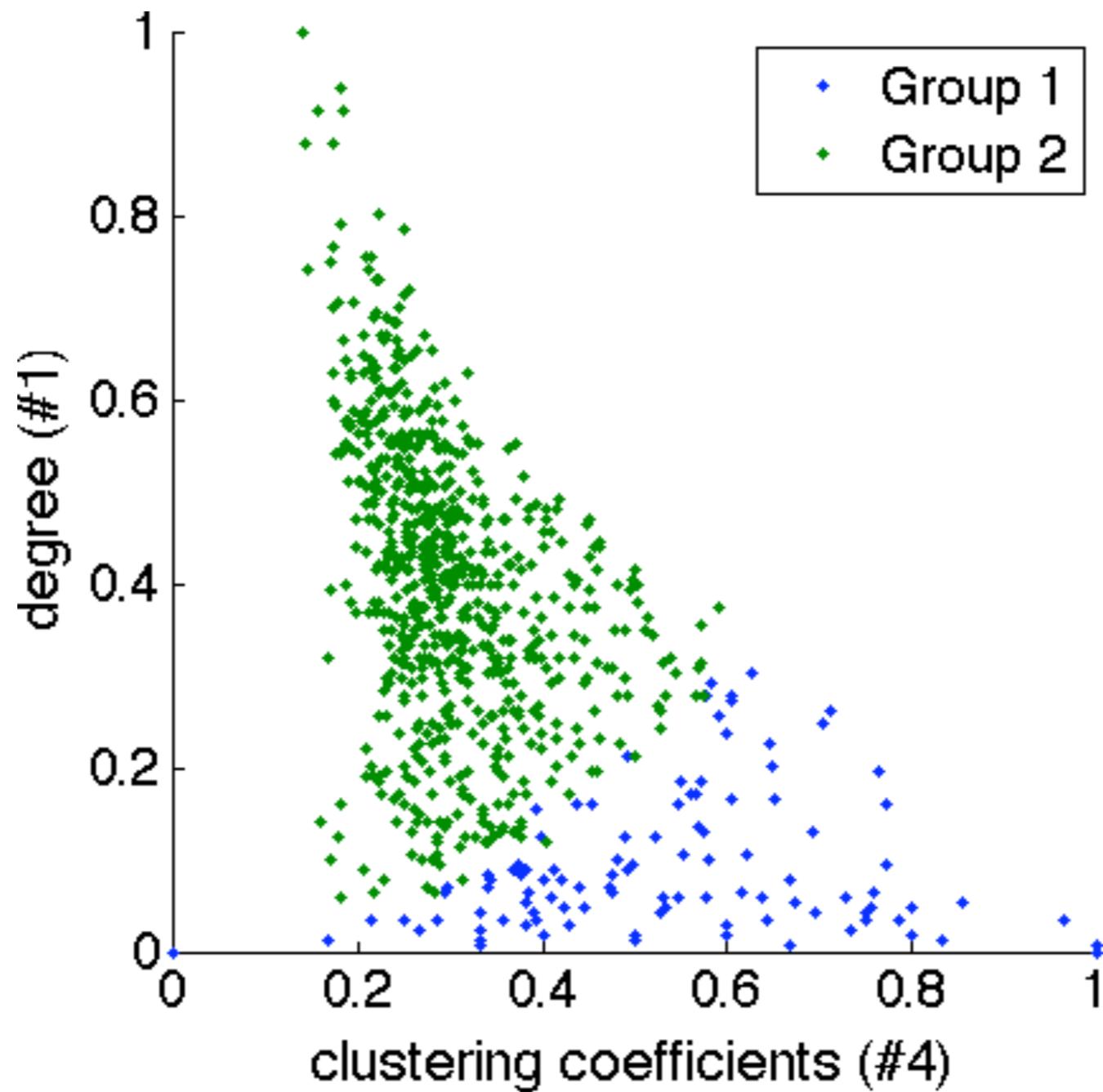


Data source: <http://www.atsweb.neu.edu/ngulbahce/pacsdata.html>
Reference: M. Herrera, D.C. Roberts and N. Gulbahce, arxiv:0904.1234

Example 4: PACS Network



A problem with k -Means Clustering



Future Work

Comprehensive benchmarking

- **As a network group detection method:**
against community detection algorithms, expectation-maximization method, ...
- **As a high-dimensional clustering method:**
against k -means clustering, unsupervised SVM, ...

Future Work

- Can we substitute human eye with 2-means clustering or unsupervised two-class SVM?
- Exploring a high-dimensional network parameter space, given only coarse & non-uniform sample points: How does dynamical properties (cascading, synchronization, etc.) depend on network structural parameters?